

AFRL-VA-WP-TP-2003-300

**COMBINING STATE DEPENDENT
RICCATI EQUATION APPROACH
WITH DYNAMIC INVERSION:
APPLICATION TO CONTROL OF
FLIGHT VEHICLES**



**Rama K. Yedavalli
Praveen Shankar
David B. Doman**

FEBRUARY 2003

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20030320 017

REPORT DOCUMENTATION PAGE				<i>Form Approved</i> OMB No. 0704-0188	
<small>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</small>					
1. REPORT DATE (DD-MM-YY) February 2003		2. REPORT TYPE Conference Paper Preprint		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE COMBINING STATE DEPENDENT RICCATI EQUATION APPROACH WITH DYNAMIC INVERSION: APPLICATION TO CONTROL OF FLIGHT VEHICLES				5a. CONTRACT NUMBER F33615-01-2-3154	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER N/A	
6. AUTHOR(S) Rama K. Yedavalli and Praveen Shankar (The Ohio State University) David B. Doman (AFRL/VACA)				5d. PROJECT NUMBER N/A	
				5e. TASK NUMBER N/A	
				5f. WORK UNIT NUMBER N/A	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) The Ohio State University Control Theory Optimization Branch (AFRL/VACA) 2036 Neil Ave. Mall Control Sciences Division Bolz Hall, Room 328 Air Vehicles Directorate Columbus, OH 43210-1276 Air Force Research Laboratory, Air Force Materiel Command Wright-Patterson Air Force Base, OH 45433-7542				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Vehicles Directorate Air Force Research Laboratory Air Force Materiel Command Wright-Patterson Air Force Base, OH 45433-7542				10. SPONSORING/MONITORING AGENCY ACRONYM(S) AFRL/VACA	
				11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S) AFRL-VA-WP-TP-2003-300	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES Proceedings to be presented in the AIAA Guidance Navigation and Control Conference, August 11, 2003, Austin, TX ©2001 AIAA. This work is copyrighted. The United States has for itself and others acting on its behalf an unlimited, paid-up, nonexclusive, irrevocable worldwide license. Any other form of use is subject to copyright restrictions.					
14. ABSTRACT (Maximum 200 Words) State Dependent Algebraic Riccati Equation (SDRE) techniques are rapidly emerging as a design method, which provides a systematic and effective means of designing nonlinear controllers, observers, and filters. This paper describes a new method of integrating the SDRE technique with the Dynamic Inversion control law that is frequently used in the design of aircraft control systems. This paper also provides an example by applying this control design technique to a reusable launch vehicle.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT: SAR	18. NUMBER OF PAGES 22	19a. NAME OF RESPONSIBLE PERSON (Monitor) David B. Doman 19b. TELEPHONE NUMBER (Include Area Code) (937) 255-8451
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			

Combining State Dependent Riccati Equation Approach with Dynamic Inversion: Application to Control of Flight Vehicles

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Abstract

State Dependent Algebraic Riccati Equation (SDRE) techniques are rapidly emerging as a design method, which provides a systematic and effective means of designing nonlinear controllers, observers and filters. This paper describes a new method of integrating the SDRE technique with the Dynamic Inversion control law that is frequently used in the design of aircraft control systems. This paper also provides an example by applying this control design technique to a reusable launch vehicle.

Introduction

There have been a number of design methodologies developed for control of nonlinear systems. The aircraft problem is one such nonlinear system to which control design techniques such as Dynamic Inversion have been applied. Lesser-known nonlinear design procedures are those that involve the state dependent Riccati equations (SDRE). The State Dependent Riccati Equation approach to nonlinear system stabilization relies on representing a nonlinear system's dynamics similar to linear dynamics, but with state dependent coefficient matrices that can be inserted into state dependent Riccati equations to generate a feedback law. Although stability of the resulting closed loop system need not be guaranteed a priori, simulation studies have shown that the method can often lead to suitable control laws.

Over the past several years various SDRE design methodologies have been successfully applied to aerospace problems. SDRE based design procedures have been used in advanced guidance law development [1,2] and in an output feedback autopilot design [3]. Additionally, SDRE design methods have been used in nonlinear filter development [4]. In [5], Ehrler and Vadali investigated the nonlinear regulator problem and showed that solving an algebraic Riccati as it evolved over time provided one means of obtaining a sub optimal solution of the infinite horizon problem. In essence the State Dependent Riccati Equation was treated as being time dependent and its state dependency was not explicitly acknowledged, addressed or analyzed. In [6], SDRE nonlinear regulation, SDRE nonlinear H_∞ , and SDRE nonlinear H_2 design methodologies were defined and the optimality, sub optimality and stability properties of SDRE nonlinear regulation was investigated.

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Overview

SDRE stabilization refers to the use of State Dependent Riccati Equations to construct nonlinear feedback control laws for nonlinear systems. The main idea is to represent the nonlinear system

$$\dot{x} = f(x) + B(x)u$$

in the form

$$\dot{x} = A(x)x + B(x)u$$

and to use the feedback

$$u = -R^{-1}(x)B^T(x)P(x)x$$

where $P(x)$ is obtained from the SDRE

$$P(x)A(x) + A^T(x)P(x) + Q(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) = 0$$

and $Q(\cdot)$ and $R(\cdot)$ are design parameters that satisfy the point wise definiteness condition

$$Q(x) > 0 \quad R(x) > 0$$

The resulting closed loop dynamics have a linear-like structure given by

$$\dot{x} = A_{cl}(x)x \text{ where}$$

$$A_{cl}(x) = A(x) - R^{-1}(x)B(x)B^T(x)P(x)$$

Simulation studies have shown that the dynamics matrix satisfies the Lyapunov Criterion for stability given by

$$P(x)A_{cl}(x) + A_{cl}^T(x)P(x) < -Q_{pd} \text{ where}$$

$$Q_{pd} > 0$$

Equations of Motion of Aircraft

$$\dot{\phi} = P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta$$

$$\dot{\theta} = Q \cos \phi - R \sin \phi$$

$$\dot{\psi} = Q \sin \phi \sec \theta + R \cos \phi \sec \theta$$

$$\dot{P} = c_1 RQ + c_2 PQ + c_3 L + c_4 N$$

$$\dot{Q} = c_5 PR - c_6 (P^2 - R^2) + c_7 M$$

$$\dot{R} = c_8 PQ - c_2 RQ + c_4 L + c_9 N$$

$$\dot{U} = RV - QW + \frac{F_x}{m} - g \sin \theta$$

$$\dot{V} = -RU + PW + \frac{F_y}{m} + g \cos \theta \sin \phi$$

$$\dot{W} = QU - PV + \frac{F_z}{m} + g \cos \theta \cos \phi$$

where

$$c_1 = [(J_y - J_z)J_z - J_{xz}^2] / \Gamma$$

$$c_2 = (J_x - J_y + J_z)J_{xz} / \Gamma$$

$$c_3 = J_z / \Gamma$$

$$c_4 = J_{xz} / \Gamma$$

$$c_5 = (J_z - J_x) / J_y$$

$$c_6 = J_{xz} / J_y$$

$$c_7 = 1 / J_y$$

$$c_8 = [(J_x - J_y)J_x + J_{xz}^2] / \Gamma$$

$$c_9 = J_x / \Gamma$$

where

$$\Gamma = J_x J_z - J_{xz}^2$$

State Dependent Linear State Space System

Example 1

$$\text{State : } x = [P \ Q \ R]$$

$$\dot{x} = \begin{bmatrix} c_2 x_2 & c_1 x_3 & 0 \\ -c_6 x_1 & 0 & c_5 x_1 + c_6 x_3 \\ 0 & c_8 x_1 & -c_2 x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 2

$$\text{State : } x = [P \ Q \ R \ \phi \ \theta \ \psi]$$

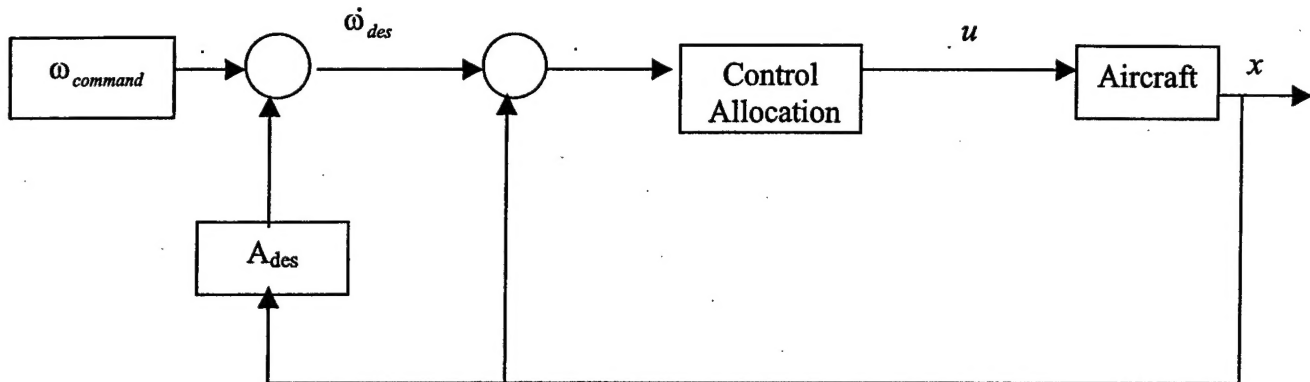
$$\dot{x} = \begin{bmatrix} c_2 x_2 & c_1 x_3 & 0 & 0 & 0 & 0 \\ -c_6 x_1 & 0 & c_5 x_1 + c_6 x_3 & 0 & 0 & 0 \\ 0 & c_8 x_1 & -c_2 x_2 & 0 & 0 & 0 \\ 1 & \sin x_5 \tan x_5 & \cos x_5 \tan x_5 & 0 & 0 & 0 \\ 0 & \cos x_4 & -\sin x_4 & 0 & 0 & 0 \\ 0 & \sin x_4 \sec x_5 & \cos x_4 \sec x_5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 3

$$\text{State : } x = [U \ V \ W \ P \ Q \ R \ \sin \theta \ \cos \theta \sin \phi \ \cos \theta \cos \phi]$$

$$\dot{x} = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ F \\ F \\ L \\ M \\ N \end{bmatrix}$$

Schematic Representation of the Method



Combined SDRE and Dynamic Inversion Control Law

Stability of Nominal System for Full State Feedback

$$\dot{x} = A(x)x + Bu$$

Riccati Based Control Law

$$u = -R^{-1}B^T P(x)x$$

Dynamic Inversion Control Law

$$u = B^{-1}(\dot{x}_{des} - A(x)x$$

$$W.K.T \quad B^{-1} = (B^T B)^{-1}$$

Let

$$\dot{x}_{des} = A_{des}(x)x$$

Then

$$u = -(B^T B)^{-1} B^T (A(x) - A_{des}(x))x$$

Comparing Equations (1) & (2)

$$R = B^T B$$

$$P(x) = A(x) - A_{des}(x)$$

The SDARE becomes

$$P(x)A(x) + A^T(x)P(x) + Q - P(x)P(x) = 0$$

Solving the above equation for $P(x)$, we can calculate

$$A_{des}(x) = A(x) - P(x)$$

Stability of Nominal System for Output Feedback

$$\dot{x} = A(x)x + Bu$$

$$w = Cx$$

$$\dot{w} = C\dot{x} = CA(x)x + CBu$$

$$\text{Let } \dot{w}_{des} = A_{des}(x)x$$

Riccati Based Control Law

$$u = -R^{-1}B^T P(x)x$$

Dynamic Inversion Control Law

$$u = (CB)^+ (\dot{w}_{des} - CA(x)x)$$

$$u = -(CB)^+ (CA(x) - A_{des}(x))x$$

$$u = -\{(CB)^T (CB)\}^{-1} (CB)^T (CA(x) - A_{des}(x))x$$

$$u = -(B^T C^T CB)^{-1} B^T C^T (CA(x) - A_{des}(x))x$$

Comparing Equations (3) & (4)

$$R = B^T C^T CB$$

$$P(x) = C^T (CA(x) - A_{des}(x))$$

The SDARE becomes

$$P(x)A(x) + A^T(x)P(x) + Q - P(x)B(B^T C^T CB)^{-1} B^T P(x) = 0$$

Solving the above equation for $P(x)$, we can calculate A_{des} using the equation

$$P(x) = C^T (CA(x) - A_{des}(x))$$

Closed Loop System

$$\dot{x} = [A(x) + B(CB)^+ (A_{des}(x) - A(x))]x = A_c(x)x$$

Verification of Stability

Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_0)$. $P(x_0)$ is the solution to the SDARE at the initial condition $\{x_0\}$.

Full State Feedback

$$P(x_0)A(x_0) + A^T(x_0)P(x_0) + Q - P(x_0)P(x_0) = 0$$

Output Feedback

$$P(x_0)A(x_0) + A^T(x_0)P(x_0) + Q - P(x_0)B(B^T C^T C B)^{-1} B^T P(x_0) = 0$$

The Closed Loop System is locally asymptotically stable if

$$P_L A_c(x) + A_c^T(x) P_L < 0$$

Application of Combined SDRE and Dynamic Inversion Control Law to Aircraft

$$\text{State : } x = [U \ V \ W \ P \ Q \ R \ \sin\theta \ \cos\theta \sin\phi \ \cos\theta \cos\phi]$$

$$\dot{x} = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ L \\ M \\ N \end{bmatrix}$$

where

$$A(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \quad \& B = \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above state space system is not completely controllable. However if we separate the $A(x)$ matrix into $A_1(x)$ and $A_2(x)$ given by

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 \end{bmatrix} \quad \text{and} \quad A_2(x) = \begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}$$

then the pair $(A_1(x), B_1)$ are completely controllable.

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 \end{bmatrix} \quad \text{and} \quad B_1 = \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \end{bmatrix}$$

Therefore we can design a control law given by

$$u = B_1^{-1}(\dot{x}_{des} - A_1(x)x_1)$$

where $x_1 = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]$
and $u = [F_x \ F_y \ F_z \ L \ M \ N]$

$$W.K.T \quad B_1^{-1} = (B_1^T B_1)^{-1}$$

Let

$$\dot{x}_{1des} = A_{1des}(x)x_1$$

Then

$$u = -(B_1^T B_1)^{-1} B_1^T (A_1(x) - A_{1des}(x))x_1$$

$A_{1des}(x)$ is calculated from the equation

$$A_{1des}(x) = A_1(x) - P(x)$$

where $P(x)$ is the solution to the Riccati Equation

$$P(x)A_1(x) + A_1^T(x)P(x) + Q - P(x)B_1(x)R^{-1}(x)B_1^T(x)P(x) = 0$$

However we know that

$$R = B_1^T B_1$$

Therefore the State Dependent Algebraic Riccati Equation reduces to

$$P(x)A_1(x) + A_1^T(x)P(x) + Q - P(x)P(x) = 0$$

The closed loop system is given by

$$\dot{x}_1 = A_1(x)x + B_1 [B_1^{-1}(A_{1des}(x) - A_1(x))x_1]$$

Therefore, we have

$$\dot{x}_1 = A_{cl}(x)x_1$$

$$\text{where } A_{cl}(x) = A_1(x) + B_1 [B_1^{-1}(A_{1des}(x) - A_1(x))]$$

Verification of Closed Loop System Stability

In the previous section we separated $A(x)$ into

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad \text{and} \quad A_2(x) = \begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}$$

The matrix $A_2(x)$ is neutrally stable if we calculate it's eigenvalues by freezing the states at each time instant. However we are more concerned about the stability of matrix $A_1(x)$ under the control 'u' that we previously discussed. The closed loop system under control 'u' is given by $A_{cl}(x)$.

Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_{10})$. $P(x_{10})$ is the solution to the SDARE at the initial condition $\{x_{10}\}$.

$$P(x_{10})A(x_{10}) + A^T(x_{10})P(x_{10}) + Q - P(x_{10})P(x_{10}) = 0$$

The closed loop system is locally asymptotically stable if

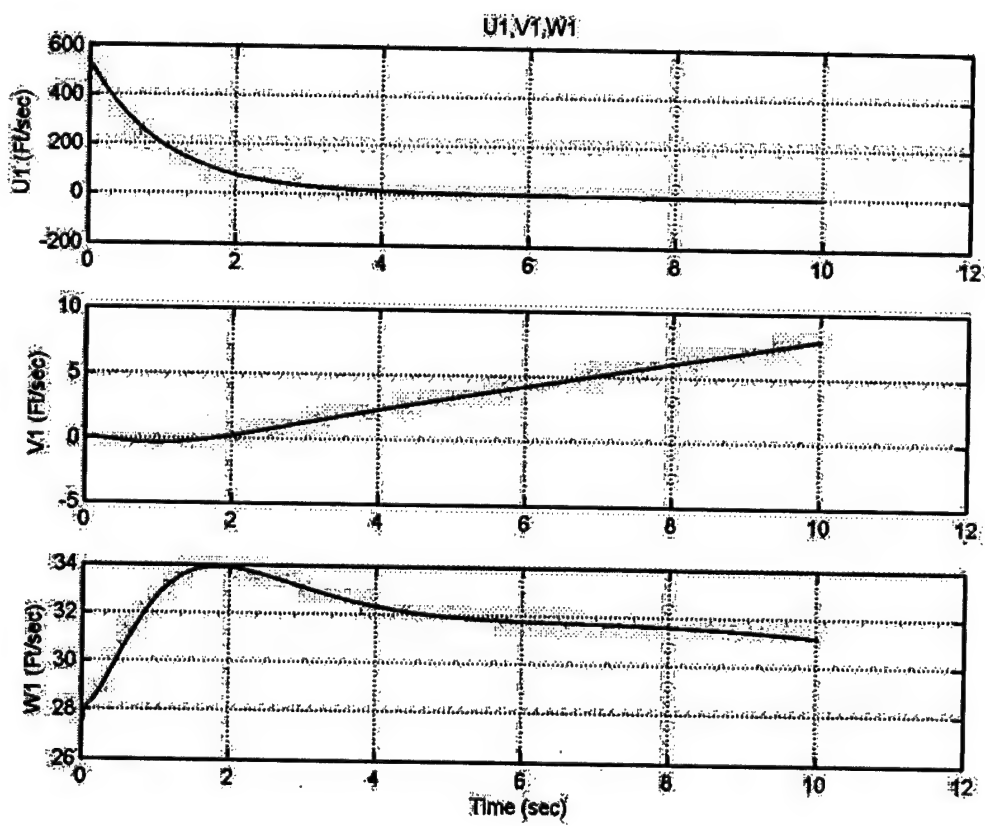
$$P_L A_{cl}(x) + A_{cl}^T(x)P_L < 0$$

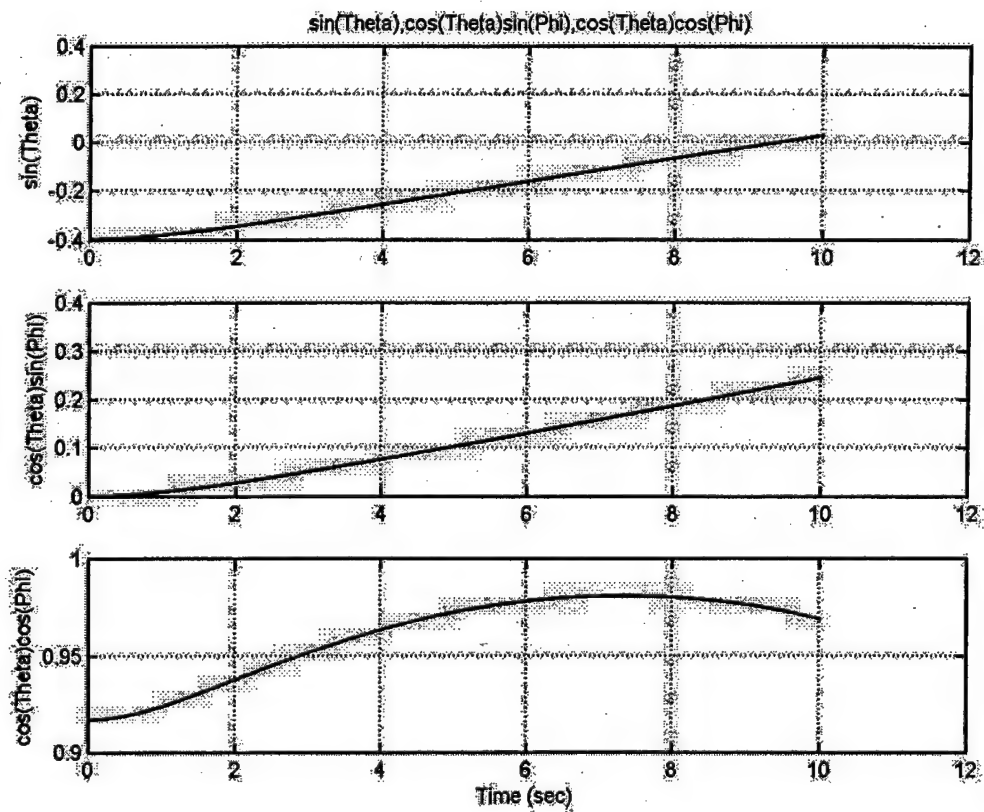
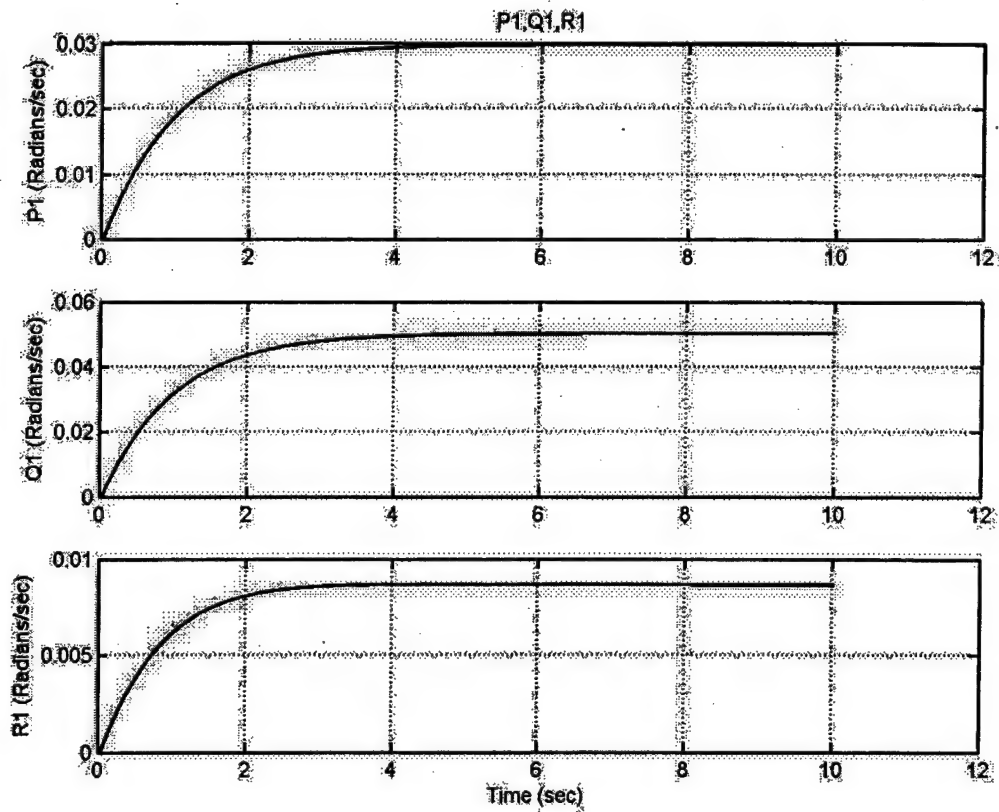
Results

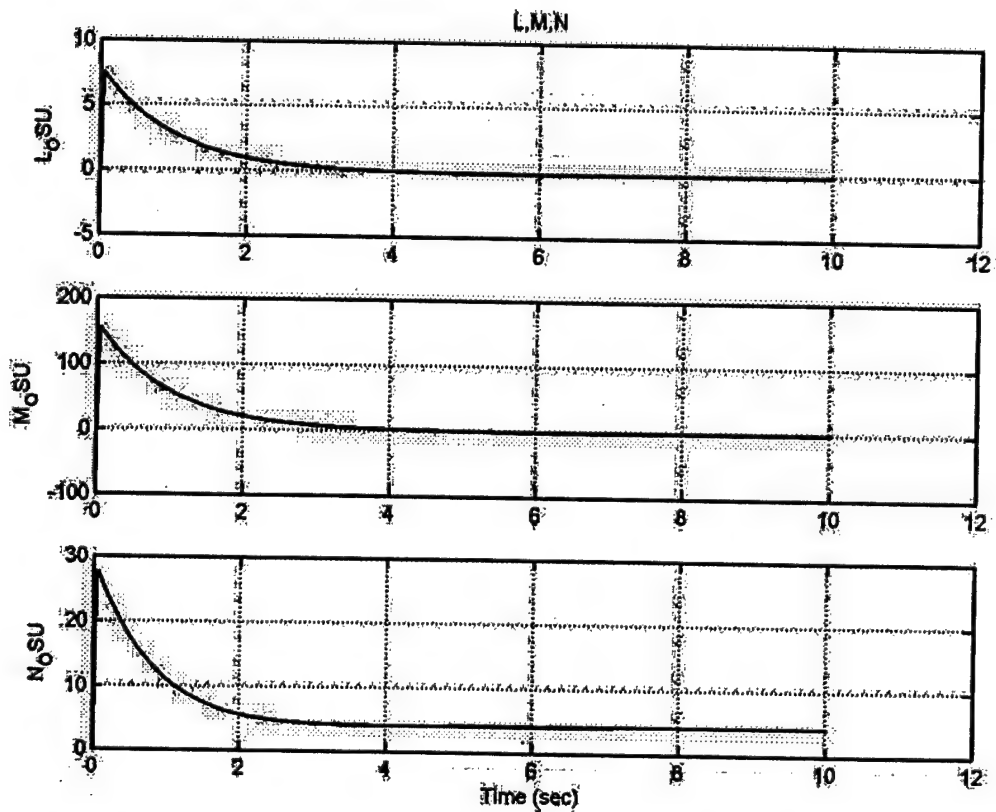
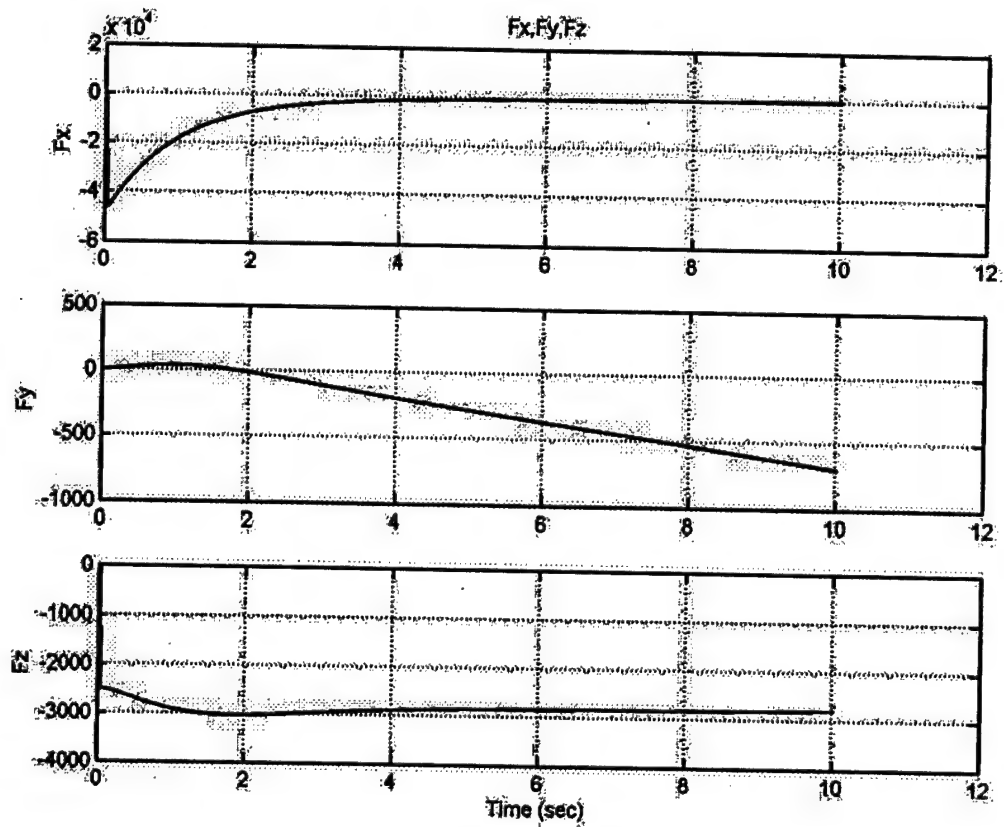
Initial Condition 1

$$U_0 = 517.5$$

$V_0 = 0$
 $W_0 = 27.5$
 $P_0 = 0$
 $Q_0 = 0$
 $R_0 = 0$







Results

Initial Condition 2

$$U_0 = 517.5$$

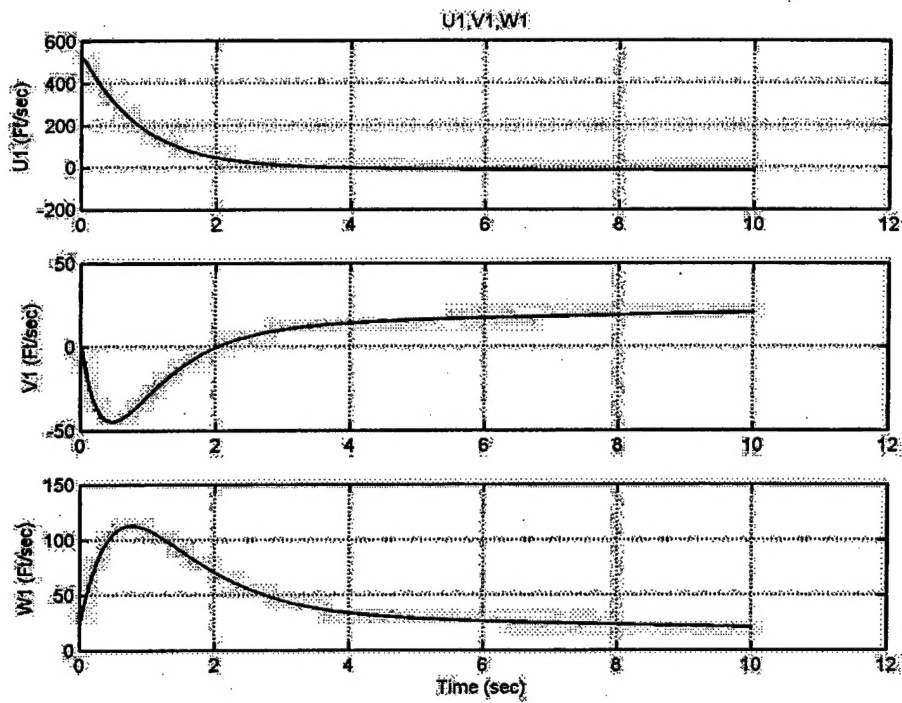
$$V_0 = 0$$

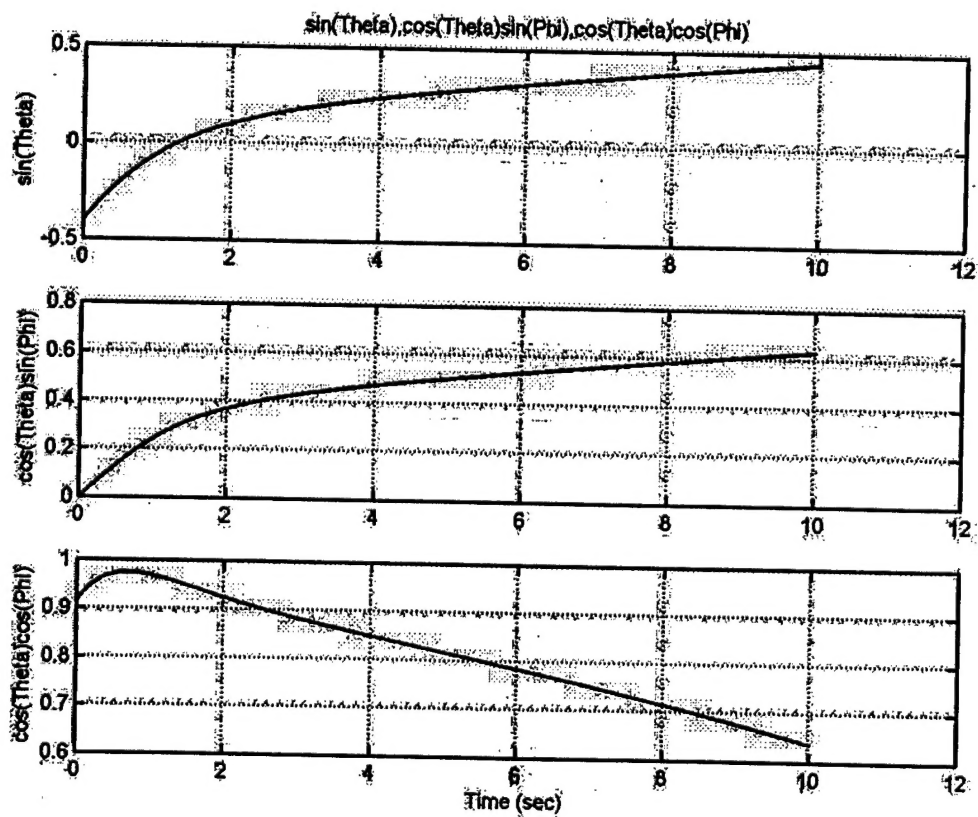
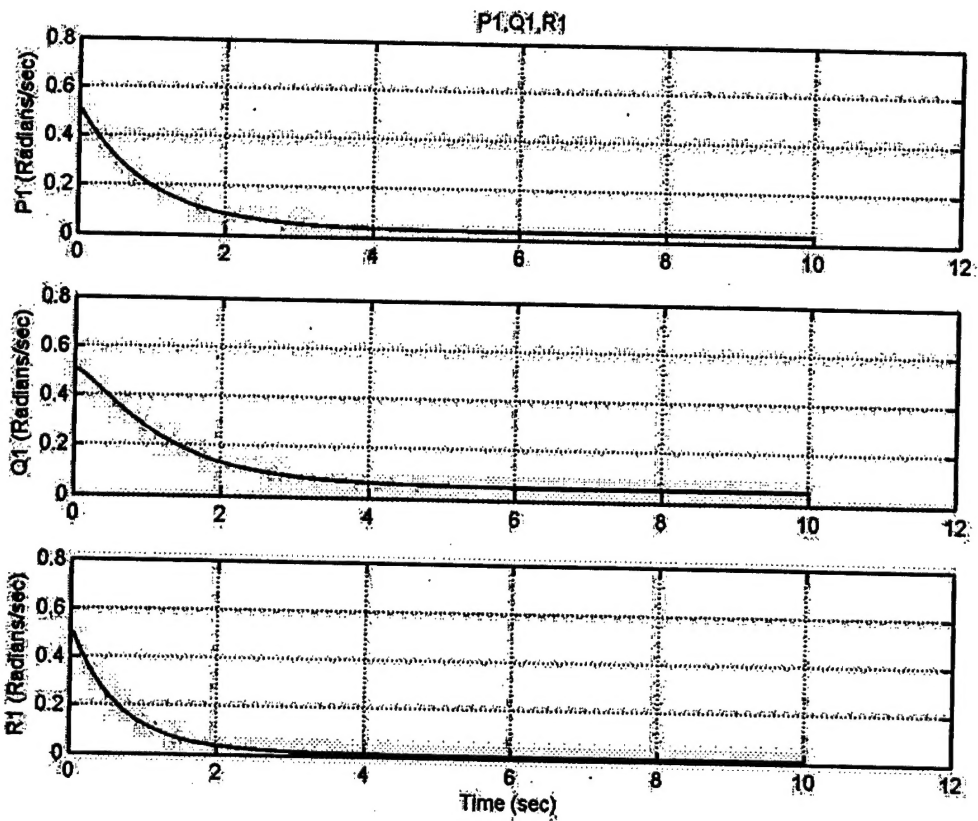
$$W_0 = 27.5$$

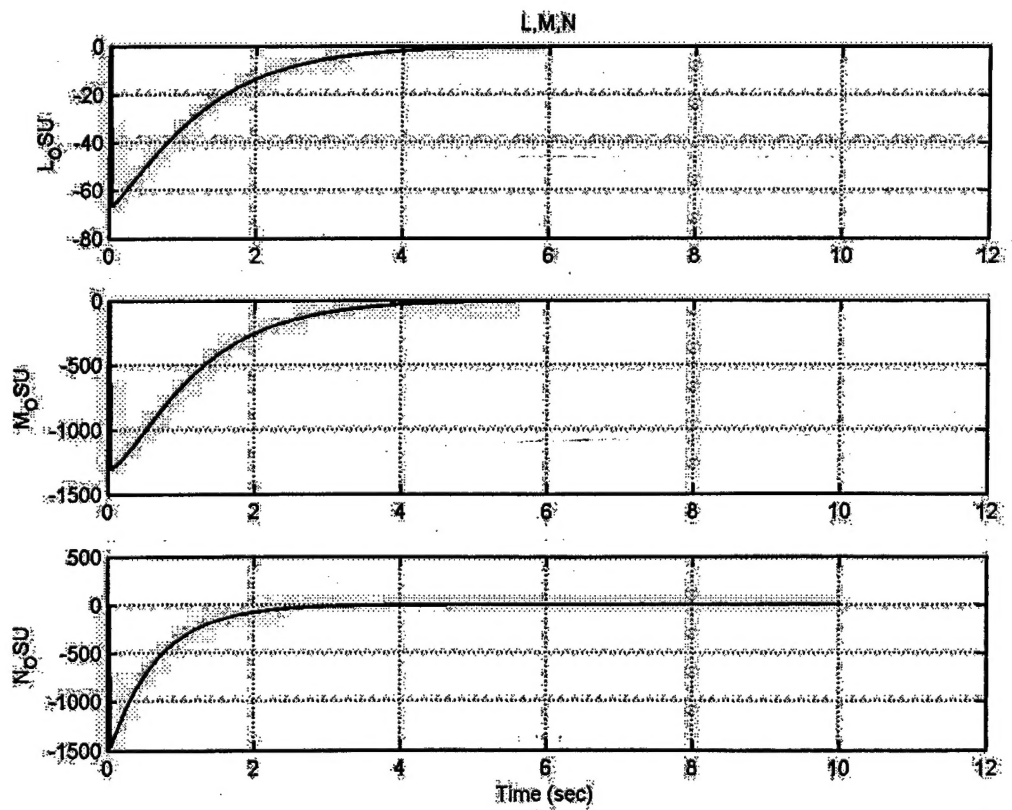
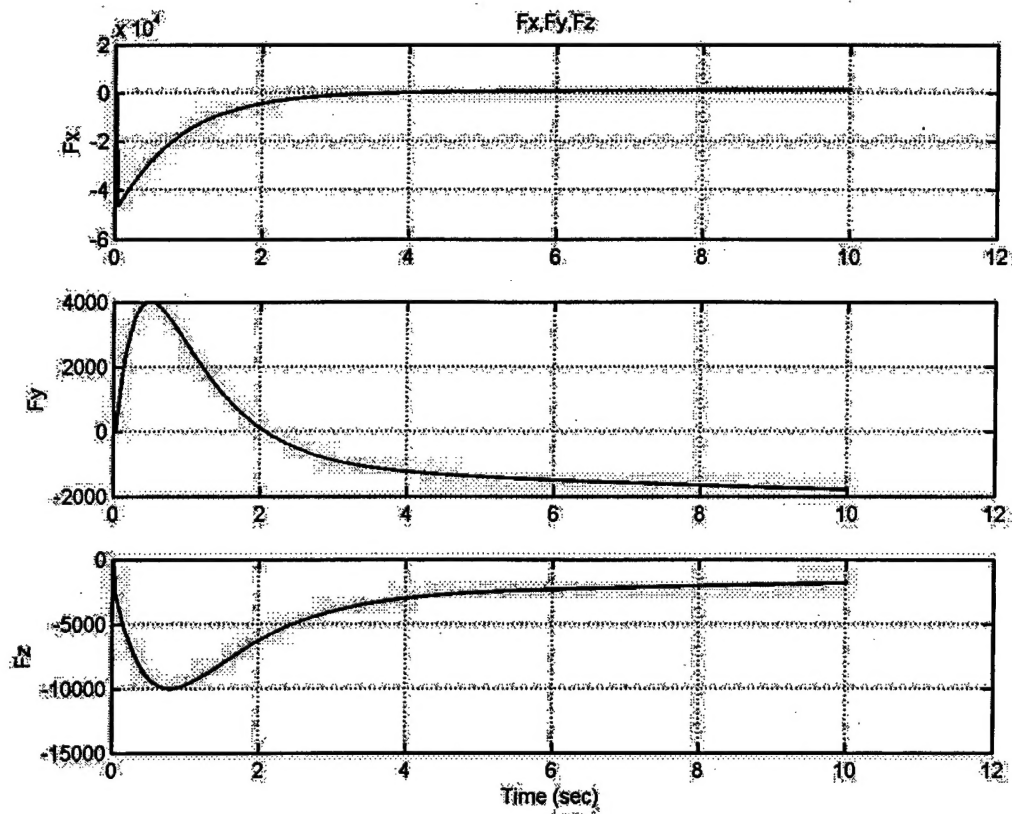
$$P_0 = 0.5$$

$$Q_0 = 0.5$$

$$R_0 = 0.5$$







Conclusions

In this paper a new control law was developed that is a combination of the existing State Dependent Riccati Equation techniques and the Dynamic Inversion control law. This control system design was then applied to the aircraft dynamics and the resulting closed loop system was shown to be stable even under change in the initial conditions.

References

- [1] J.R Cloutier "*Adaptive matched augmented proportional navigation.*" Presented at AIAA Missile Sciences Conference, November 1994.
- [2] J.R. Cloutier "*Time-to-go-less guidance with cross-channel couplings.*" Proceedings of the AIAA Missile sciences Conference, Monterey, CA, December 1996.
- [3] C.P. Mracek and J.R. Cloutier "*Missile longitudinal autopilot design using the state dependent Riccati equation method.*" Proceedings of the International Conference on Nonlinear Problems in Aviation and Aerospace, May 1996.
- [4] C.P Mracek, J.R Cloutier, and C.N. D'Souza "*A new technique for nonlinear estimation.*" Proceedings of the IEEE Conference on Control Applications, Dearborn, MI, September 1996.
- [5] D. Ehrler and S.R. Vadali "*Examination of the optimal nonlinear regulator problem.*" Proceedings of the AIAA Guidance, Navigation and Control Conference, Minneapolis, MN, August 1988.
- [6] J.R. Cloutier, C.N. D'Souza, and C.P. Mracek "*Nonlinear regulation and nonlinear H_∞ control via the state dependent Riccati equation technique; part 1, theory; part 2, examples.*" Proceedings of the International Conference on Nonlinear Problems in Aviation and Aerospace, May 1996.
- [7] J.R. Cloutier "*State Dependent Riccati Equation Techniques: An Overview*" Proceedings of American Control Council Conference, June 1997
- [8] Zhihua Qu and J.R. Cloutier "*A New Sub-optimal Control for Cascaded Nonlinear Systems*" Proceedings of American Control Council Conference, June 2000
- [9] J.R Cloutier and D.T. Stansbery "*The Capabilities and Art of State-Dependent Riccati Equation-Based Design*" Proceedings of American Control Council Conference, June 2002
- [10] "*Dynamics of Flight Stability and Control*" Bernard Etkin and Lloyd Duff Reid, John Wiley and Sons, 3rd Ed.
- [11] "*Aircraft Control and Simulation*" Brian L. Stevens and Frank L. Lewis, John Wiley and Sons
- [12] "*Robust Control of Nonlinear Uncertain Systems*" Zhihua Qu, John Wiley and Sons
- [13] D.B Doman and A.D Ngo "*Dynamic Inversion Based Adaptive/Reconfigurable Control of the X-33 on Ascent*" Journal of Guidance Control and Dynamics, 2002